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Many structures can be approximated as a straight beam or as a collection of straight beams. For this reason, the analysis of stresses and deflections in a beam is an important and useful topic. This section covers shear force and bending moment in beams, shear and moment diagrams, stresses in beams, and a table of common beam deflection formulas. Contents For a beam to remain in static equilibrium when external loads are applied to it, the beam must be constrained. Constraints are defined at single points along the beam, and the boundary condition at that point determines the nature of the constraint. The boundary condition indicates whether the beam is fixed (restrained from motion) or free to move in each direction. For a 2-dimensional beam, the directions of interest are the x-direction (axial direction), y-direction (transverse direction), and rotation. For a constraint to exist at a point, the boundary condition must indicate that at least one direction is fixed at that point. Common boundary conditions are shown in the table below. For each boundary condition, the table indicates whether the beam is fixed or free in each direction at the point where the boundary condition is defined. Boundary ConditionDirection Axial (X)Transverse (Y)Rotation FreeFreeFreeFree FixedFixedFixedFixed PinnedFixedFixedFree Guided along XFreeFixedFixed Guided along YFixedFreeFixed Roller along XFreeFixedFree Roller along YFixedFreeFree If the boundary condition indicates that the beam is fixed in a specific direction, then an external reaction in that direction can exist at the location of the boundary condition. For example, if a beam is fixed in the y-direction at a specific point, then a transverse (y) external reaction force may develop at that point. Likewise, if the beam is fixed against rotation at a specific point, then an external reaction moment may develop at that point. Based on the above discussion, we can see that a fixed boundary condition can develop axial and transverse reaction forces as well as a moment. Likewise, we see that a pinned boundary condition can develop axial and transverse reaction forces, but it cannot develop a reaction moment. Notice the Free boundary condition in the table above. This boundary condition indicates that the beam is free to move in every direction at that point (i.e., it is not fixed or constrained in any direction). Therefore, a constraint does not exist at this point. This highlights the subtle difference between a constraint and a boundary condition. A boundary condition indicates the fixed/free condition in each direction at a specific point, and a constraint is a boundary condition in which at least one direction is fixed. Shear Force and Bending Moment To find the shear force and bending moment over the length of a beam, first solve for the external reactions at each constraint. For example, the cantilever beam below has an applied force shown as a red arrow, and the reactions are shown as blue arrows at the fixed boundary condition. The external reactions should balance the applied loads such that the beam is in static equilibrium. After the external reactions have been solved for, take section cuts along the length of the beam and solve for the internal reactions at each section cut. (The reaction forces and moments at the section cuts are called internal reactions because they are internal to the beam.) An example section cut is shown in the figure below: When the beam is cut at the section, either side of the beam may be considered when solving for the internal reactions. The side that is selected does not affect the results, so choose whichever side is easiest. In the figure above, the side of the beam to the right of the section cut was selected. The selected side is shown as the blue section of beam, and section shown in grey is ignored. The internal reactions at the section cut are shown with blue arrows. The reactions are calculated such that the section of beam being considered is in static equilibrium. Sign Convention The signs of the shear and moment are important. The sign is determined after a section cut is taken and the reactions are solved for the portion of the beam to one side of the cut. The shear force at the section cut is considered positive if it causes clockwise rotation of the selected beam section, and it is considered negative if it causes counter-clockwise rotation. The bending moment at the section cut is considered positive if it compresses the top of the beam and elongates the bottom of the beam (i.e., if it makes the beam "smile"). Based on this sign convention, the shear force at the section cut for the example cantilever beam in the figure above is positive since it causes clockwise rotation of the selected section. The moment is negative since it compresses the bottom of the beam and elongates the top (i.e., it makes the beam "frown"). The figure below shows the standard sign convention for shear force and bending moment. The forces and moments on the left are positive, and those on the right are negative. Check out our beam calculator based on the methodology described here. Calculates stresses and deflections in straight beams Builds shear and moment diagrams Can specify any configuration of constraints, concentrated forces, and distributed forces Shear and Moment Diagrams The shear force and bending moment throughout a beam are commonly expressed with diagrams. A shear diagram shows the shear force along the length of the beam, and a moment diagram shows the bending moment along the length of the beam. These diagrams are typically shown stacked on top of one another, and the combination of these two diagrams is a shear-moment diagram. Shear-moment diagrams for some common end conditions and loading configurations are shown within the beam deflection tables at the end of this page. An example of a shear-moment diagram is shown in the following figure: General rules for drawing shear-moment diagrams are given in the table below. All of the rules in this table are demonstrated in the figure above. Shear Diagram Moment Diagram Point loads cause a vertical jump in the shear diagram. The direction of the jump is the same as the sign of the point load. Uniform distributed loads result in a straight, sloped line on the shear diagram. The slope of the line is equal to the value of the distributed load. The shear diagram is horizontal for distances along the beam with no applied load. The shear at any point along the beam is equal to the slope of the moment at that same point: The moment diagram is a straight, sloped line for distances along the beam with no applied load. The slope of the line is equal to the value of the shear. Uniform distributed loads result in a parabolic curve on the moment diagram. The maximum/minimum values of moment occur where the shear line crosses zero. The moment at any point along the beam is equal to the area under the shear diagram up to that point: Bending Stresses in Beams The bending moment, M, along the length of the beam can be determined from the moment diagram. The bending moment at any location along the beam can then be used to calculate the bending stress over the beam's cross section at that location. The bending moment varies over the height of the cross section according to the flexure formula below: where M is the bending moment at the location of interest along the beam's length, Ic is the centroidal moment of inertia of the beam's cross section, and y is the distance from the beam's neutral axis to the point of interest along the height of the cross section. The negative sign indicates that a positive moment will result in a compressive stress above the neutral axis. The bending stress is zero at the beam's neutral axis, which is coincident with the centroid of the beam's cross section. The bending stress increases linearly away from the neutral axis until the maximum values at the extreme fibers at the top and bottom of the beam. The maximum bending stress occurs at the extreme fiber of the beam and is calculated as: where c is the centroidal distance of the cross section (the distance from the centroid to the extreme fiber). If the beam is asymmetric about the neutral axis such that the distances from the neutral axis to the top and to the bottom of the beam are not equal, the maximum stress will occur at the farthest location from the neutral axis. In the figure below, the tensile stress at the top of the beam is larger than the compressive stress at the bottom. The section modulus of a cross section combines the centroidal moment of inertia, Ic, and the centroidal distance, c: The benefit of the section modulus is that it characterizes the bending resistance of a cross section in a single term. The section modulus can be substituted into the flexure formula to calculate the maximum bending stress in a cross section: Check out our beam calculator based on the methodology described here. Calculates stresses and deflections in straight beams Builds shear and moment diagrams Can specify any configuration of constraints, concentrated forces, and distributed forces Shear Stresses in Beams The shear force, V, along the length of the beam can be determined from the shear diagram. The shear force at any location along the beam can then be used to calculate the shear stress over the beam's cross section at that location. The average shear stress over the cross section is given by: The shear stress varies over the height of the cross section, as shown in the figure below: The shear stress is zero at the free surfaces (the top and bottom of the beam), and it is maximum at the centroid. The equation for shear stress at any point located a distance y1 from the centroid of the cross section is given by: where V is the shear force acting at the location of the cross section, Ic is the centroidal moment of inertia of the cross section, and b is the width of the cross section. These terms are all constants. The Q term is the first moment of the area bounded by the point of interest and the extreme fiber of the cross section: Shear stresses for several common cross sections are discussed in the sections below. Shear Stresses in Rectangular Sections The distribution of shear stress along the height of a rectangular cross section is shown in the figure below: The first moment of area at any given point y1 along the height of the cross section is calculated by: The maximum value of Q occurs at the neutral axis of the beam (where y1 = 0): The shear stress at any given point y1 along the height of the cross section is calculated by: where Ic = bh3/12 is the centroidal moment of inertia of the cross section. The maximum shear stress occurs at the neutral axis of the beam and is calculated by: where A = bh is the area of the cross section. We can see from the previous equation that the maximum shear stress in the cross section is 50% higher than the average stress V/A. Shear Stresses in Circular Sections A circular cross section is shown in the figure below: The equations for shear stress in a beam were derived using the assumption that the shear stress along the width of the beam is constant. This assumption is valid at the centroid of a circular cross section, although it is not valid anywhere else. Therefore, while the distribution of shear stress along the height of the cross section cannot be readily determined, the maximum shear stress in the section (occurring at the centroid) can still be calculated. The maximum value of first moment, Q, occurring at the centroid, is given by: The maximum shear stress is then calculated by: where b = 2r is the diameter (width) of the cross section, Ic = r4/4 is the centroidal moment of inertia, and A = r2 is the area of the cross section. Shear Stresses in Circular Tube Sections A circular tube cross section is shown in the figure below: The maximum value of first moment, Q, occurring at the centroid, is given by: The maximum shear stress is then calculated by: where b = 2 (ro ri) is the effective width of the cross section, Ic = (ro4 ri4) / 4 is the centroidal moment of inertia, and A = (ro2 ri2) is the area of the cross section. Shear Stresses in I-Beams The distribution of shear stress along the web of an I-Beam is shown in the figure below: The equations for shear stress in a beam were derived using the assumption that the shear stress along the width of the beam is constant. This assumption is valid over the web of an I-Beam, but it is invalid for the flanges (specifically where the web intersects the flanges). However, the web of an I-Beam takes the vast majority of the shear force (approximately 90% - 98%, according to Gere), and so it can be conservatively assumed that the web carries all of the shear force. The first moment of the area of the web of an I-Beam is given by: The shear stress along the web of the I-Beam is given by: where tw is the web thickness and Ic is the centroidal moment of inertia of the I-Beam: The maximum value of shear stress occurs at the neutral axis (y1 = 0), and the minimum value of shear stress in the web occurs at the outer fibers of the web where it intersects the flanges y1 = hw/2): PDH Classroom offers a continuing education course based on this beam analysis reference page. This course can be used to fulfill PDH credit requirements for maintaining your PE license. Now that you've read this reference page, earn credit for it! Beam Deflection Tables The tables below give equations for the deflection, slope, shear, and moment along straight beams for different end conditions and loadings. You can find comprehensive tables in references such as Gere, Lindeburg, and Shigley. However, the tables below cover most of the common cases. Cantilever Beams Cantilever, End Load @ x = L @ x = L Cantilever, Intermediate Load (0 x a) (a x L) @ x = L (0 x a) (a x L) V = +F (0 x a) V = 0 (a x L) M = F (a x) (0 x a) M = 0 (a x L) Cantilever, Uniform Distributed Load @ x = L @ x = L Cantilever, Triangular Distributed Load @ x = L @ x = L Cantilever, End Moment @ x = L @ x = L Simply Supported Beams Simply Supported, Intermediate Load (0 x a) For a b: @ (0 x a) @ x = 0 @ x = L V1 = +Fb / L (0 x a) V2 = Fa / L (a x L) Simply Supported, Center Load (0 x L/2) @ x = L/2 (0 x L/2) @ x = 0 @ x = L V1 = +F / 2 (0 x L/2) V2 = F / 2 (L/2 x L) Simply Supported, 2 Loads at Equal Distances from Supports (0 x a) (a x L a) @ x = L/2 (0 x a) (a x L a) @ x = 0 @ x = L V1 = +F (0 x a) V2 = F (L a x L) Mmax = Fa (a x L a) Simply Supported, Uniform Distributed Load @ x = L/2 @ x = 0 @ x = L V1 = +wL / 2 @ x = 0 V2 = wL / 2 @ x = L Simply Supported, Moment at Each Support @ x = L/2 @ x = 0 @ x = L Simply Supported, Moment at One Support @x = L (1 3/3) @ x = 0 @ x = L Simply Supported, Center Moment (0 x L/2) (0 x L/2) @ x = 0 @ x = L M = M0x / L (0 x L/2) Mmax = M0 / 2 @ x = L/2 Fixed-Fixed Beams Fixed-Fixed, Center Load (0 x L/2) @ x = L/2 V1 = +F / 2 (0 x L/2) V2 = F / 2 (L/2 x L) M = F (4x L) / 8 (0 x L/2) M1 = M3 = FL / 8 @ x = 0 & x = L M2 = +FL / 8 @ x = L/2 Fixed-Fixed, Uniform Distributed Load @ x = L/2 V1 = +wL / 2 @ x = 0 V2 = wL / 2 @ x = L M = w (6Lx 6x2 L2) / 12 M1 = M3 = wL2 / 12 @ x = 0 & x = L M2 = wL2 / 24 @ x = L/2 Subscribe to receive occasional updates on the latest improvements: References Budynas-Nisbett, "Shigley's Mechanical Engineering Design," 8th Ed. Gere, James M., "Mechanics of Materials," 6th Ed. Lindeburg, Michael R., "Mechanical Engineering Reference Manual for the PE Exam," 13th Ed. "Stress Analysis Manual," Air Force Flight Dynamics Laboratory, October 1986. So far we have discussed what a force is and how it causes normal stresses and normal strains on an object. We have also discussed what a bending moment is, but what effects would a bending moment have on a beam? How would it cause the beam to deflect and would it cause stresses on a beam? To answer these questions we will use Euler-Bernoulli Beam Bending Theory (click for useful info). Euler-Bernoulli Beam Bending Theory Equation Without going too much into the derivation, Euler-Bernoulli beam bending theory gives the following equation for how a bending moment effects a beam. A full derivation of the equation can be found here. $$EI \frac{d^{2}v}{dx^{2}} = M $$ Where \(E\) is the Youngs modulus of the beam, \(I\) is the Second Moment of Area of a beam, \(v\) is the vertical deflection of the beam and \(x\) is the distance along a beam. \(M\) is the moment at the x-distance along the beam that is being examined and so will typically be a different value at each location. Using Euler-Bernoulli beam bending theory also allows the relationship between shear force, \(V\), and moment to be expressed as: $$V = \frac{dM}{dx}$$ I.e. the shear force is equal to the rate of change of moment. This is a useful identity for sanity checking bending moment and shear force diagrams: For a linear moment behaviour (typically due to applied point loads) the applied shear force will be constant. For a quadratic moment behaviour (typically due to applied uniformly distributed load) the shear force will change linearly. If the shapes of the bending moment and shear diagram don't match the above rules of thumb, then further checking of the bending and shear force equations is probably needed. Euler-Bernoulli Beam Bending Theory Assumptions It is important to understand the assumptions that are made in Euler-Bernoulli beam bending theory, the main assumptions are: Plane sections remain plane This is a complicated way of saying that as a beam deflects, sections that started as perpendicular to the neutral axis before bending will remain perpendicular to the neutral axis after bending. This assumes there are no significant shear deformations within the beam, this is generally a valid assumption unless the beam is very deep compared to its span. A beam is typically defined as a deep beam if the span to depth ratio is less than 2. Angles of deformations remain small This assumes that the angle of deformation is sufficiently small to allow the small angle approximation of \(sin \theta \approx \theta \) i.e. \(\theta \approx \frac{dv}{dx}\) to be applied. The beam is formed of a material that is homogeneous (same material properties in all locations) and isotropic (same material properties in all directions). In addition, it is assumed the beam behaves elastically. Figure 1: Plane sections remaining plane during bending. I.e. section that started perpendicular to the neutral axis, remain perpendicular to the neutral axis during bending. Euler-Bernoulli beam bending can be seen as simplification of the later Timoshenko beam theory, this more advanced beam theory allows for shear deformations, rotational effects and provides a better model for thick beams. Elastic Beam Bending Equations Euler-Bernoulli beam bending theory gives rise to the elastic beam bending equations below, these are incredibly useful equations for structural analysis of beams: $$ \frac{M}{I} =\frac{E}{R} =\frac{\sigma}{y} $$ Where, M is the applied moment, I is the second moment of area of the beam, E is the Youngs modulus of the beam, R is the radius of curvature of the neutral axis, \(\sigma\) is the resulting stress at a point on the beam cross section, y is the distance from the neutral axis to the location on the cross section where you would like to find your value of stress. The above equation can be rearranged to link applied moment, the section properties of a beam and the stress resulting from the applied moment: $$\sigma= \frac{My}{I}$$ Often the section properties of the beam \(\frac{I}{y}\) are grouped into a single section property called the section modulus: $$Z= \frac{I}{y}$$ Where Z is the section modulus, this is parameter and how it is used is discussed in more detail in a separate article. To Top of Page Regardless of their shape, length, supports or loads, all beams bend slightly. However, the extent of thisbending depends hugely on these factors. This is where beam theory comes in, to calculate the bending & deflection of beams.There is a vertical reaction at both endsDeflection at each end is zeroAngle at each end is not necessarily zeroMaximum deflection is in the centre (when no loads are applied)One end has two vertical reaction forces next to one another, pointing in opposite directionsDeflection at fixed end is zeroDeflection at cantilevered end is always maximumAngle at fixed end is not necessarily zeroDeflection at built-in end is zeroAngle at built-in end is zeroMaximum deflection is always at cantilevered endThe built-in end provided both a moment and a reaction forceDeflection at both ends is zeroAngle at both ends is zeroMaximum deflection is in the middle (when no loads are applied)There is a moment and reaction force at both endsRemember: a built in end provides a moment!Often, shear force and bending moment diagrams need to be drawn in order to find the maximumbending moment in a beam. To do this, the beam is split into regions areas between two changesin load, and each region is sectioned in the middleAsimply supported beam with no loads on it has only one region. Add a point load in the middle,and there are two regions, one on either side, etc.The example above has four regions:From the left end to , with the distributed load acting throughoutFrom to the end of the distributed loadFrom the end of the distributed load to From to the right endNote that the arrow labelled only represents the total weight of the distributed load, nothingchanged in terms of load at this point, so there is no new region.It is important to stick to the sign convention whenever solving problems that involve sectioningbeams:Now lets add some numbers to the example above and go through the calculations, step by step:1. Taking moments about the left side to find the reaction forces 2. Section in the middle of region 1 3. Section in the middle of region 2 4. Section in the middle of region 3 5. Substitute in the region boundariesLeft LimitRight Limit Left LimitRight LimitThe moment at the right end must also equal zero.Now we can draw the shear force and bending moment diagrams:We can clearly see the maximum bending moment is -32 Nm. This is needed to calculate themaximum stress in the beam, and the deflections.Often, we need to know the maximum stress in a beam, to predict where it will break. This is givenby the equation: is the maximum bending moment is the distance from the neutral axis of the stress is the second moment of areaThe units of second moment of area is m.The second moment of area is a property of a cross-section. For rectangles and circles it is givenas:Second moment of areaSecond moments of area can be added and subtracted if the cross sections lie on the same neutral axis.If they do not lie on the same neutral axis, the parallel axis theorem must be used:Takethe T-section as an example:First, the overall neutral axis must be found: Then, the overall second moment of area can be found: This equation applies for a T-section, where y is the distance between the overall neutral axis and y isbelow the overall neutral axes.A measure of the deflection of a beam is the radius of curvature, R. This brings together everythingimportant about beam theory, through the fundamental equation: is the maximum stress is the distance from the neutral axis is the bending moment is the second moment of area is the Youngs modulus is the radius of curvatureOften, the second moment of area and Youngs Modulus are combined to give a constantthe structural rigidity, .Rearranging this equation gives , the second derivative of deflection: Integrating this gives the angle of deflection, also known as slope: Integrate again gives the deflection of the beam: There are two ways of solving for deflection, Macaulays method and superposition:This defines the bending moment as a step function in terms of Macaulay brackets (pointybrackets). Each Macaulay bracket is turned on or off, depending on whether or not it applies to thepart of the beam we are looking at.Macaulay brackets are integrated as single entities, not using reverse chain rule.To find the Macaulay function, section the beam in the last region, using Macaulay brackets forthe distances.Using this beam setup, sectioned at the end, we get: The first term in the brackets only applies when The second term in the brackets only applies when The third term in the brackets only applies whenIntegrating the above once gives the slope: Integrating again gives the deflection when : When integrating the function for the bending moment, the two constants of integration can befound from the types of support, e.g. at a built-in support, both slope and deflection are zero,whereas at a simple support, only deflection is zero (see Supports).Therefore: Alternatively, you can treat the different loads on a beam individually, find the deflection causedby each one, and then add these all together. To do this, use standard results:Simply Supported Beams:Moment-Held Beams:Built-in Cantilevered Beams:Built-in at both ends:Remember that if you want to find the end deflection of a load that does not go all the way to theend, you need to extrapolate the deflection by multiplying the slope by the length to the end of thebeam: The bending moment is the internal torque experienced by a beam when subjected to external loads. Understanding how this parameter varies along the axis of the beam is essential in designing safe structures. This article tackles the bending moment and how it affects the flexural stress in a beam. To comprehend this phenomenon better, it is important to first understand the basics of beam structures and loading types. A beam is generally used to support various structures, including the floor of a building, the deck of a bridge, or the wing of an aircraft. It is a horizontal structural element that primarily bears vertical loads. These loads result in internal forces and moments that cause the beam to deform. There are two types of loading in beam structures: distributed loads and concentrated loads. Distributed loads are applied uniformly along the length of the beam, while concentrated loads are focused on specific points. The bending moment at any point along the length of the element is determined by the distribution and magnitude of the applied loads, as well as the geometry and support conditions of the element. Advance in Excel with engineering-focused training that equips you with the skills to streamline projects and accelerate your career. Mathematically, bending moment is the algebraic sum of the moments applied to the beam at any point along its length. It is calculated by multiplying the force acting on the beam by the distance from the applied force to the point at which it is being measured. Bending moments can be positive or negative depending on the specific conditions and forces acting on the structural element. By convention, a positive bending moment occurs when the applied forces and moments cause the structural element to bend or flex in a way that the top of the element is in compression and the bottom is in tension. In a simply supported beam, a positive bending moment usually occurs when a load or force is applied above the beam, creating a sagging or concave shape. On the other hand, a negative bending moment occurs when the applied forces and moments cause the top of the structural element to be in tension and the bottom to be in compression. In a simply supported beam, a negative bending moment typically occurs when a load or force is applied below the beam, creating a humped or convex shape. These are illustrated in the diagram below: In general, the bending moment varies from point to point along the axis of the beam. In order to design a structurally safe beam, it is important to determine the maximum bending moment that is experienced by the beam and where it occurs. One way to do this is by creating a bending moment diagram. A bending moment diagram is a graphical representation of the variation in bending moment along the length of the beam. It is typically plotted along the horizontal axis, which represents the length of the structural element, while the vertical axis represents the magnitude of the bending moment. One method commonly used to draw bending moment diagrams is the superposition method. This involves breaking down the complex load system on the beam into simpler components and subsequently combining their effects. Shear and moment diagrams for each single load can be drawn separately, and then summed to obtain the overall diagrams. As an example, the diagram below shows the shear and bending moment diagrams of a beam with a uniformly distributed loading. In addition to superposition, the area method is another technique for constructing bending moment diagrams. This method is based on the concept that the area under the shear force diagram between two points on a beam equals the change in bending moment between those two points. When analyzing bending moment diagrams, it is crucial to consider the sign conventions. Generally, clockwise moments are considered negative, and counterclockwise moments are considered positive. Shear forces that generate clockwise moments are designated as negative, while those causing counterclockwise moments are assigned positive values. Bending moment diagrams are typically used to identify areas of the structure that experience the most significant forces. This helps in distributing loads and designing reinforcement at critical points to prevent structural failure. Flexural stress is a type of mechanical stress that occurs when a beam is subjected to a bending moment, which causes the object to deform and bend. Bending stress can be tensile or compressive, depending on whether the material on one side of the bending is stretched or compressed relative to the other side. In beams subjected to just bending, a linear relationship between bending moment and the flexural stress is observed, as expressed in the following equation: Where: = bending stress [Pa] M = bending moment [N-m] y = distance from the neutral axis to the point at which the stress is being calculated [m] I = moment of inertia [m4] Bending stress is a critical factor in the design and analysis of structures and beams, as it helps determine whether a given structure or component can safely support the loads and moments to which it will be subjected. Many structures can be approximated as a straight beam or as a collection of straight beams. For this reason, the analysis of stresses and deflections in a beam is an important and useful topic. This section covers shear force and bending moment in beams, shear and moment diagrams, stresses in beams, and a table of common beam deflection formulas. Contents For a beam to remain in static equilibrium when external loads are applied to it, the beam must be constrained. Constraints are defined at single points along the beam, and the boundary condition at that point determines the nature of the constraint. The boundary condition indicates whether the beam is fixed (restrained from motion) or free to move in each direction. For a 2-dimensional beam, the directions of interest are the x-direction (axial direction), y-direction (transverse direction), and rotation. For a constraint to exist at a point, the boundary condition must indicate that at least one direction is fixed at that point. Common boundary conditions are shown in the table below. 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Likewise, we see that a pinned boundary condition can develop axial and transverse reaction forces, but it cannot develop a reaction moment. Notice the Free boundary condition in the table above. This boundary condition indicates that the beam is free to move in every direction at that point (i.e., it is not fixed or constrained in any direction). Therefore, a constraint does not exist at this point. This highlights the subtle difference between a constraint and a boundary condition. A boundary condition indicates the fixed/free condition in each direction at a specific point, and a constraint is a boundary condition in which at least one direction is fixed. Shear Force and Bending Moment To find the shear force and bending moment over the length of a beam, first solve for the external reactions at each constraint. For example, the cantilever beam below has an applied force shown as a red arrow, and the reactions are shown as blue arrows at the fixed boundary condition. The external reactions should balance the applied loads such that the beam is in static equilibrium. After the external reactions have been solved for, take section cuts along the length of the beam and solve for the internal reactions at each section cut. (The reaction forces and moments at the section cuts are called internal reactions because they are internal to the beam.) An example section cut is shown in the figure below: When the beam is cut at the section, either side of the beam may be considered when solving for the internal reactions. The side that is selected does not affect the results, so choose whichever side is easiest. In the figure above, the side of the beam to the right of the section cut was selected. The selected side is shown as the blue section of beam, and section shown in grey is ignored. The internal reactions at the section cut are shown with blue arrows. The reactions are calculated such that the section of beam being considered is in static equilibrium. Sign Convention The signs of the shear and moment are important. The sign is determined after a section cut is taken and the reactions are solved for the portion of the beam to one side of the cut. The shear force at the section cut is considered positive if it causes clockwise rotation of the selected beam section, and it is considered negative if it causes counter-clockwise rotation. The bending moment at the section cut is considered positive if it compresses the top of the beam and elongates the bottom of the beam (i.e., if it makes the beam "smile"). Based on this sign convention, the shear force at the section cut for the example cantilever beam in the figure above is positive since it causes clockwise rotation of the selected section. The moment is negative since it compresses the bottom of the beam and elongates the top (i.e., it makes the beam "frown"). The figure below shows the standard sign convention for shear force and bending moment. 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Gere, James M., "Mechanics of Materials," 6th Ed. Lindeburg, Michael R., "Mechanical Engineering Reference Manual for the PE Exam," 13th Ed. "Stress Analysis Manual," Air Force Flight Dynamics Laboratory, October 1986. PreviousNext Bending moments are produced by transverse loads applied to beams. The simplest case is the cantilever beam , widely encountered in balconies, aircraft wings, diving boards etc. The bending moment acting on a section of the beam, due to an applied transverse force, is given by the product of the applied force and its distance from that section. It thus has units of Nm. It is balanced by the internal moment arising from the stresses generated. This is given by a summation of all of the internal moments acting on individual elements within the section. These are given by the force acting on the element (stress times area of element) multiplied by its distance from the neutral axis, y . Balancing the external and internal moments during the bending of a cantilever beam Therefore, the bending moment, M , in a loaded beam can be written in the form \[M = \int {y(\sigma dA)} \] The concept of the curvature of a beam, , is central to the understanding of beam bending. The figure below, which refers now to a solid beam, rather than the hollow pole shown in the previous section, shows that the axial strain, , is given by the ratio y / R . Equivalently, 1/R (the "curvature",) is equal to the through-thickness gradient of axial strain. It follows that the axial stress at a distance y from the Neutral axis of the beam is given by =Ey Relation between the radius of curvature, R, beam curvature, , and the strains within a beam subjected to a bending moment. The bending moment can thus be expressed as \[M = \int {y(E\kappa ydA)} = \kappa E\int {{y^2}} dA\] This can be presented more compactly by defining I (the second moment of area, or "moment of inertia") as \[I = \int\limits_0^{{y_{\max }}} {} {y^2}{\rm{d}}A\] The units of I are m 4 . The value of I is dependent solely on the beam sectional shape. Click here to see how I is calculated for two simple shapes. The moment can now be written as M=EI These equations allow the curvature distribution along the length of a beam (ie its shape), and the stress distribution within it, to be calculated for any given set of applied forces. The following simulation implements these equations for a user-controlled beam shape and set of forces. The 3-point bending and 4-point bending loading configurations in this simulation are SYMMETRICAL, with the upward forces, denoted by arrows, outside of the downward force(s), denoted by hooks A fruitful approach to designing beams which are both light and stiff is to make them hollow. Calculation of the second moment of area for hollow beams is very straightforward, since it is obtained by simply subtracting the I of the missing section from that of the overall section. For example, that for a cylindrical tube is given by \[I = {I_{{\rm{complete\;setion}}}} - {I_{{\rm{missing\;setion}}}} = \frac{{\pi {D^4}}}{{64}} - \frac{{\pi {d^4}}}{{64}}\] Many structures can be approximated as a straight beam or as a collection of straight beams. For this reason, the analysis of stresses and deflections in a beam is an important and useful topic. This section covers shear force and bending moment in beams, shear and moment diagrams, stresses in beams, and a table of common beam deflection formulas. Contents For a beam to remain in static equilibrium when external loads are applied to it, the beam must be constrained. Constraints are defined at single points along the beam, and the boundary condition at that point determines the nature of the constraint. The boundary condition indicates whether the beam is fixed (restrained from motion) or free to move in each direction. For a 2-dimensional beam, the directions of interest are the x-direction (axial direction), y-direction (transverse direction), and rotation. For a constraint to exist at a point, the boundary condition must indicate that at least one direction is fixed at that point. Common boundary conditions are shown in the table below. For each boundary condition, the table indicates whether the beam is fixed or free in each direction at the point where the boundary condition is defined. Boundary ConditionDirection Axial (X)Transverse (Y)Rotation FreeFreeFreeFree FixedFixedFixedFixed PinnedFixedFixedFree Guided along XFreeFixedFixed Guided along YFixedFreeFixed Roller along XFreeFixedFree Roller along YFixedFreeFree If the boundary condition indicates that the beam is fixed in a specific direction, then an external reaction in that direction can exist at the location of the boundary condition. For example, if a beam is fixed in the y-direction at a specific point, then a transverse (y) external reaction force may develop at that point. Likewise, if the beam is fixed against rotation at a specific point, then an external reaction moment may develop at that point. Based on the above discussion, we can see that a fixed boundary condition can develop axial and transverse reaction forces as well as a moment. Likewise, we see that a pinned boundary condition can develop axial and transverse reaction forces, but it cannot develop a reaction moment. Notice the Free boundary condition in the table above. This boundary condition indicates that the beam is free to move in every direction at that point (i.e., it is not fixed or constrained in any direction). Therefore, a constraint does not exist at this point. This highlights the subtle difference between a constraint and a boundary condition. A boundary condition indicates the fixed/free condition in each direction at a specific point, and a constraint is a boundary condition in which at least one direction is fixed. Shear Force and Bending Moment To find the shear force and bending moment over the length of a beam, first solve for the external reactions at each constraint. For example, the cantilever beam below has an applied force shown as a red arrow, and the reactions are shown as blue arrows at the fixed boundary condition. The external reactions should balance the applied loads such that the beam is in static equilibrium. After the external reactions have been solved for, take section cuts along the length of the beam and solve for the internal reactions at each section cut. (The reaction forces and moments at the section cuts are called internal reactions because they are internal to the beam.) An example section cut is shown in the figure below: When the beam is cut at the section, either side of the beam may be considered when solving for the internal reactions. The side that is selected does not affect the results, so choose whichever side is easiest. In the figure above, the side of the beam to the right of the section cut was selected. The selected side is shown as the blue section of beam, and section shown in grey is ignored. The internal reactions at the section cut are shown with blue arrows. The reactions are calculated such that the section of beam being considered is in static equilibrium. Sign Convention The signs of the shear and moment are important. The sign is determined after a section cut is taken and the reactions are solved for the portion of the beam to one side of the cut. The shear force at the section cut is considered positive if it causes clockwise rotation of the selected beam section, and it is considered negative if it causes counter-clockwise rotation. The bending moment at the section cut is considered positive if it compresses the top of the beam and elongates the bottom of the beam (i.e., if it makes the beam "smile"). Based on this sign convention, the shear force at the section cut for the example cantilever beam in the figure above is positive since it causes clockwise rotation of the selected section. The moment is negative since it compresses the bottom of the beam and elongates the top (i.e., it makes the beam "frown"). The figure below shows the standard sign convention for shear force and bending moment. 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