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Join our visual experts and special guests for an info-packed hour of insights to elevate your next video project. Tune in on June 24 at 11am ET.Register NowHow can financial brands set themselves apart through visual storytelling? Our experts explain how.Learn MoreThe Motorsport Images Collections captures events from 1895 to today's most recent coverage.Discover The CollectionWant to give your brand videos a cinematic edge? Join our visual experts and special guests for an info-packed hour of insights to elevate your next video project. Tune in on June 24 at 11am ET.Register NowHow can financial brands set themselves apart through visual storytelling? Our experts explain how.Learn MoreThe Motorsport Images Collections captures events from 1895 to today's most recent coverage.Discover The Collection Learning Outcomes Find the vertex, axis of symmetry, [latex]y[/latex]-intercept, and/or minimum or maximum value of a quadratic function in the vertex form [latex]f(x)=a{(x-h)}^{2}+k[/latex]. Graph quadratic functions in vertex form. Figure 1. An array of satellite dishes. (credit: Matthew Colvin de Valle, Flickr) Curved antennas, such as the ones shown in the photo, are commonly used to focus microwaves and radio waves to transmit television and telephone signals, as well as satellite and spacecraft communication. The cross-section of the antenna is in the shape of a parabola, which can be described by a quadratic function. Quadratic Functions Below are three forms of the same quadratic function. Vertex Form: [latex]f(x)=-2(x-3)^2+2[/latex] Intercept Form: [latex]f(x)=-2(x-2)(x-4)[/latex] General Form: [latex]f(x)=-2x^2+12x-16[/latex] What do all of these functions have in common? What makes all of the above functions QUADRATIC functions? A Quadratic Function is a polynomial function of degree [latex]2[/latex]. The verb quadrare in Latin means "to make square." The quadratic term [latex]-2x^2[/latex] can be read "negative two multiplied by [latex]x[/latex] to the second power" or more simply, "negative two [latex]x[/latex] squared." The graph of a quadratic function is a curved called a parabola. Quadratic Functions Non-Quadratic Functions [latex]f(x)=x^2[/latex] [latex]y=3x+2[/latex] [latex]y=5x-4x^2-1[/latex] [latex]f(x)=x^4-3x^3+5x^2-4x+1[/latex] [latex]g(x)=4(x+1)(x-3)[/latex] [latex]y=|x^2-4|[/latex] [latex]y=-(x+4)^2-5[/latex] [latex]g(x)=(x^3-9)[/latex] Recognize characteristics of parabolas The graph of a quadratic function is a U-shaped curve called a parabola. In this section, we will learn how to graph parabolas. One important feature of the graph is that it has an extreme point, called the vertex. If the parabola opens upward, the vertex represents the lowest point on the graph, or the minimum value of the quadratic function. If the parabola opens downward, the vertex represents the highest point on the graph, or the maximum value. In either case, the vertex is the turning point on the graph. The graph of a parabola is symmetric with respect to a vertical line drawn through the vertex, called the axis of symmetry. These features are illustrated in Figure 2. Figure 2 The [latex]y[/latex]-intercept is the point at which the parabola crosses the [latex]y[/latex]-axis. The value of [latex]y[/latex] at which [latex]x = 0[/latex]; [latex](0,\text{__})[/latex]. The [latex]x[/latex]-intercept(s) are the point(s) at which the parabola touches or crosses the [latex]x[/latex]-axis. If they exist, the [latex]x[/latex]-intercepts represent the zeros, or roots, of the quadratic function, the values of [latex]x[/latex] at which [latex]y= 0[/latex]; [latex](\text{__}, 0)[/latex]. The intercepts are illustrated in Figure 2. Determine the vertex, axis of symmetry, [latex]x[/latex]-intercepts (if any), and [latex]y[/latex]-intercept of the parabola shown in Figure 3. Figure 3 A parent function is the simplest form of the type of function given. Figure 4 is the graph of [latex]y=x^2[/latex], which is the parent function of quadratic functions. Figure 4 Understand how the graph of a parabola is related to its quadratic function The vertex form of a quadratic function presents in the form [latex]f\left(x\right)=a{\left(x-h\right)}^{2}+k[/latex] where [latex]\left(h,\text{ }k\right)[/latex] is the vertex. How do the [latex]a[/latex], [latex]h[/latex], and [latex]k[/latex] change the parabola? Click on "Desmos" in the lower right corner of the graph below. That will open the graph in a new tab where you can explore the ways the [latex]a[/latex], [latex]h[/latex], and [latex]k[/latex] change the graph. Use the sliders on the left side to change the look of the parabola. As you move each slider, notice how it is changing the graph from the original graph of [latex]y=x^2[/latex]. How is the [latex]a[/latex] changing the graph? (Hint: explore what happens when [latex]a[/latex] is negative, when [latex]a[/latex] is more than 0 but less than 1, and when [latex]a[/latex] is positive.) How is the [latex]h[/latex] changing the graph? (Make sure to move the sliders in both positive and negative directions.) How is the [latex]k[/latex] changing the graph? (Make sure to move the sliders in both positive and negative directions.) Graphing Quadratic Functions of the form [latex]f(x)={x}^{2}+k[/latex] Let's begin by looking at the graph of quadratic functions of the form [latex]f(x)={x}^{2}+k[/latex] such as the functions [latex]\require{color}\color{MidnightBlue}{g(x)=x^2+3}[/latex] and [latex]\color{ForestGreen}{h(x)=x^2-4}[/latex]. We will compare these two functions to the parent function [latex]\color{BrickRed}{f(x)=x^2}[/latex]. Figure 5 The vertical shift of the graph depends on the value of [latex]k[/latex] in the function [latex]f(x)=x^2+k[/latex]. When [latex]k[/latex] is positive, the graph is shifted up [latex]k[/latex] units. When [latex]k[/latex] is negative, the graph is shifted down [latex]|k|[/latex] units. Notice in Figure 5 that the graph of [latex]\color{MidnightBlue}{g(x)=x^2+3}[/latex] is identical to the graph of [latex]\color{BrickRed}{f}[/latex] except that it is shifted vertically up 3 units and that the graph of [latex]\color{ForestGreen}{h(x)=x^2-4}[/latex] is identical to the graph of [latex]\color{BrickRed}{f}[/latex] except that it is shifted vertically down 4 units. Graphing Quadratic Functions of the form [latex]f(x)=(x-h)^2[/latex] We now will look at the graph of quadratic functions of the form [latex]f(x)=(x-h)^2[/latex]. Our goal is to determine the effect adding or subtracting a real number, [latex]h[/latex] from [latex]x[/latex], has on the parent function [latex]f(x)=x^2[/latex]. Let's look at the functions [latex]\color{MidnightBlue}{g(x)=(x-3)^2}[/latex] and [latex]\color{forestGreen}{h(x)=(x+5)^2}[/latex]. We will compare these two functions to the parent function [latex]\color{BrickRed}{f(x)=x^2}[/latex]. Figure 6 The horizontal shift of the graph depends on the value of [latex]h[/latex] in the function [latex]f(x)=(x-h)^2[/latex]. When [latex]h[/latex] is subtracted from [latex]x[/latex], the graph is shifted right [latex]h[/latex] units. When [latex]h[/latex] is added to [latex]x[/latex], the graph is shifted left [latex]h[/latex] units. Notice in Figure 6 that the graph of [latex]\color{MidnightBlue}{g(x)=(x-3)^2}[/latex] is identical to the graph of [latex]\color{BrickRed}{f}[/latex] except that it is shifted horizontally right [latex]3[/latex] units and that the graph of [latex]\color{forestGreen}{h(x)=(x+5)^2}[/latex] is identical to the graph of [latex]\color{BrickRed}{f}[/latex] except that it is shifted horizontally left [latex]5[/latex] units. Combining Horizontal and Vertical Shifts Let's now look at combining horizontal and vertical shifts that are of the form [latex]f(x)=(x-h)^2+k[/latex]. Graph the function [latex]g(x)={(x+2)}^{2}-3[/latex] using transformations and label the vertex. Graphing Quadratic Functions of the form [latex]f(x)=a{x}^{2}[/latex] We now will look at the graph of quadratic functions of the form [latex]f(x)=a{x}^{2}[/latex]. Our goal is to determine the effect that multiplying by a non-zero real number, [latex]a[/latex] has on the parent function [latex]f(x)=x^2[/latex]. Let's look at the functions [latex]\color{MidnightBlue}{g(x)=2{x}^{2}}[/latex] and [latex]\color{forestGreen}{h(x)=\dfrac{1}{2}{x}^{2}}[/latex]. We will compare these two functions to the parent function [latex]\color{BrickRed}{f(x)=x^2}[/latex]. Does multiplying by [latex]2[/latex] or [latex]\dfrac{1}{2}[/latex] cause the vertex to move to a different location? Does multiplying by [latex]2[/latex] or [latex]\dfrac{1}{2}[/latex] change the shape of the parabola? Figure 7 In Figure 7, we can see that the vertex remained in the same place, at [latex](0,0)[/latex], but the shape of the parabola changed. The graph of [latex]\color{MidnightBlue}{g(x)=2{x}^{2}}[/latex] is the same as the [latex]\color{BrickRed}{f}[/latex] function but stretched vertically by a factor of [latex]2[/latex] (the blue graph appears narrower than [latex]\color{BrickRed}{f(x)=x^2}[/latex]). As [latex]a[/latex] gets larger, the parabola gets narrower, or vertically stretched. What happens to the graph when [latex]a[/latex] is between zero and one ([latex]0 Figure 8 The magnitude of [latex]a[/latex] indicates the stretch of the graph. If [latex]|a|>1[/latex], there is a vertical stretch and the graph appears to become narrower. If [latex]|a|Graphing Quadratic Functions of the form [latex]f(x)=a(x-h)^2+k[/latex] What is [latex]a[/latex], [latex]h[/latex], and [latex]k[/latex] in figure 9? The quadratic function of the graph below written in vertex form is [latex]y=-3{\left(x+2\right)}^{2}+4[/latex]. In this form, [latex]a=-3,\text{ }h=-2[/latex], and [latex]k=4[/latex]. Since [latex]a Figure 9 [latex]-3(x+2)^2+4[/latex] If [latex]k>0[/latex], the graph shifts upward, whereas if [latex]k0[/latex], so the graph is shifted [latex]4[/latex] units upward. If [latex]h>0[/latex], the graph shifts toward the right and if [latex]h1[/latex]. The magnitude of [latex]a[/latex] indicates the stretch of the graph. When [latex]|a|>1[/latex] there is a vertical stretch causing the graph to become narrower. Compare the graph of [latex]f(x)=\dfrac{1}{2}{(x-4)}^{2}-6[/latex] to the parent function [latex]f(x)=x^2[/latex] by stating the transformations. Find the vertex, axis of symmetry, two other points on graph, and then graph the function. Determine if the vertex is a maximum or minimum. A polynomial function of degree two is called a quadratic function. The graph of a quadratic function is a parabola. A parabola is a U-shaped curve that can open either up or down. The axis of symmetry is the vertical line passing through the vertex. The [latex]x[/latex]-intercepts, are the points at which the parabola crosses the [latex]x[/latex]-axis. The [latex]y[/latex]-intercept is the point at which the parabola crosses the [latex]y-[/latex]axis. Vertex form is useful to easily identify the vertex of a parabola. A quadratic function's minimum or maximum value is given by the [latex]y[/latex]-value of the vertex. axis of symmetry a vertical line drawn through the vertex of a parabola around which the parabola is symmetric; it is defined by [latex]x=-\frac{b}{2a}[/latex]. quadratic function a polynomial function of degree 2. The graph of a quadratic function is a curve called a parabola. vertex form of a quadratic function the function that describes a parabola, written in the form [latex]f\left(x\right)=a{\left(x-h\right)}^{2}+k[/latex], where [latex]\left(h,\text{ }k\right)[/latex] is the vertex. vertex the point at which a parabola changes direction, corresponding to the minimum or maximum value of the quadratic function RadfordMathematics.com Online Mathematics Book A quadratic equation has no real solution if the value of the discriminant is negative.When we find the roots of a quadratic equation, we usually come across one or two real solutions, but it is also possible that we don't get any real solutions. In this article, we will discuss quadratic equations in detail and what happens when they don't have real solutions, along with numerical examples.There are three different ways to tell whether the solution to a given quadratic equation is real or not, and these methods are calculating the discriminant, looking at the graph, and looking at the coefficients.Calculating the DiscriminantThe easiest way to tell that the given quadratic equation or function has no real roots is to calculate the value of the discriminant. If it is negative, then the quadratic equation does not have any real solutions. If the quadratic equation is given as $ax^{2}+bx +c = 0$, then we can write the standard form of the quadratic formula as:$x = \dfrac{-b \pm \sqrt{b^{2}-4ac }}{2a}$In this formula, the term $b^{2}- 4ac$ is called discriminant, denoting it as "D". The quadratic equation can have three solutions depending on the value of "D".1. The solution is real if "D" is > 0. This means we have two distinct solutions.2. If "D" is equal to zero, then we have a single real solution.3. If "D" < 0, we will have two complex solutions. In this case, we do not get a real solution.So, for a quadratic equation with complex solutions, the value of $b^{2}-4ac$ will be less than zero or $b^{2}< 4ac$. Let us compare examples for each case of the discriminant.$x^{2}+ 3x + 5$$x^{2}-2x + 1$$x^{2}-3x + 2$$a = 1$, $b = 3$ and $c = 5$$a = 1$, $b = -2$ and $c = 1$$a = 1$, $b = -3$ and $c = 2$$b^{2}= 3^{2}= 9$$b^{2}= (-2)^{2}= 4$$b^{2}= (-3)^{2}= 9$$4ac = 4(1)(4) = 20$$4ac = 4(1)(1) = 4$4ac = 4(1)(2) = 8$b^{2}< 4ac$$b^{2}= 4ac$ and $D = 0$$b^{2}> 4ac$ and $D > 0$Hence, this quadratic equation has complex roots.Hence, this quadratic equation has one real root.Hence, this quadratic equation will have two real roots.The roots of the equation are $x = -1.5 + 1.6658i$ and $-1.5 – 1.6658i$The root of the equation is $x =1$The roots of the equation are $x = 2,1$You can verify these solutions by putting the values of a, b, and c in the quadratic formula. From the above table, we can deduce that whenever $b^{2}< 4ac$, we will only get complex roots.Looking at the GraphThe second method to tell whether the quadratic equation or function has any real solution or not is by looking at the graph of the function or equation. The graph of any quadratic equation will be a parabola or bell-shaped, and we know the most important feature of a parabola is its vertex.The shape of the vertex of the parabola depends upon "a"; if the value of "a" is negative, then the shape of the vertex is like a mountain top or peak. If the value of "a" is positive, then the shape is like a valley bottom at the bottom of the mountain. A quadratic equation graph with complex solutions will not touch the x-axis.The parabola can be completely above or below the x-axis if the equation has complex solutions. When the value of $a0$, the parabola will be above the x-axis. Let us draw the graph for three equations discussed in the previous section.For the equation $x^{2}+ 3x + 5$, we know all solutions are complex, and as we can see below, the graph is above the x-axis as "a" is greater than zero. The graph is not touching the x-axis, so if you are provided with a graph and you are asked to tell whether the function has real solutions or not, you can instantly tell if the graph is not touching the x-axis then it will only have complex solutions.For the equation $x^{2}-2x +1$, we know the value of the discriminant is equal to zero; for this case, the parabola peak will always touch the x-axis. It will not go across the x-axis; the peak will land on the x-axis, as shown in the figure below.For the equation $x^{2}-3x +2$, we know the value of the discriminant is greater than zero; for this case, the parabola peak will cross the x-axis. If the value of $a > 0$, then the peak value or mountain top will be going down the x-axis and if the value of $a < 0$, then the peak value or mountain top will be above the x-axis. We show the graph below.Looking at the CoefficientsIn the third method, we look at the coefficients of the given equation. Remember the equation should be given in the normal quadratic equation form as $ax^{2}+bx + c = 0$.We can only use this method in special circumstances, for example, when we are not provided with the value of "b" or the value of "b" is equal to zero. Furthermore, the sign of the coefficients "a" and "c" must also be the same. For $b = 0$, if both "c" and "a" are positive then $\dfrac{c}{a}$ is positive and -\dfrac{c}{a} is negative and similarly if both "c" and "a" are negative then $\dfrac{c}{a}$ is positive and $-\dfrac{c}{a}$ is negative. In both cases, taking the square root will give us two complex solutions.Let us take an example of the quadratic equation $x^{2}+ 6 = 0$, we can see that in this equation $a = 1$, $b = 0$ and $c = 6$. The roots for given equation are $2.449i$ and $-2.449i$.Similarly, if we take the example of quadratic equation $-3x^{2}- 6 = 0$, we can see that in this equation $a = -3$, $b = 0$ and $c = -6$. The roots for the given equations are $1.41i$ and $-1.41i$. So, we can see that when signs of coefficients "a" and "c" were the same and b was equal to zero, we only get complex solutions.Does the Quadratic Equation Always Have a Solution?Yes, the quadratic equation will always have a solution that can either be complex or real. The quadratic equation can have a maximum of 2 real solutions. So the real solution for a quadratic equation can be 0,1, or 2, depending upon the type of quadratic equation. Similarly, the quadratic equations' complex roots can be 2 or zero. We can summarize the roots of the quadratic equation as follows:• When the value of the discriminant is positive, then we will have two real solutions.• When the value of the discriminant is equal to zero, we will have a single real solution.• When the value of the discriminant is negative, we will have two complex solutions.Examples of Quadratic EquationsLet us now study examples by solving quadratic equations having real or complex solutions. We will study no real solution quadratic equation examples and real solution quadratic equation examples.Example 1: Solve the quadratic equation $x^{2}+ 2x + 2$Solution:We know for the given quadratic equation the value of $a =1$, $b = 2$ and $c =24$The value of $b^{2}= 2^{2}= 4$$4ac = 4 (1)(2) = 8$$b^{2}- 4ac = 4 – 8 = -4$.As the value of discriminant is less than zero, then this equation will only have complex solutions. Let us put the value of a, b and c in quadratic formula and solve for the roots to verify.$x = \dfrac{-2 \pm \sqrt{-4 }}{2(1)}$$x = -1 \pm 1i$Example 2: Will the quadratic equation $-2x^{2}+4 = 0$ have real roots or not?Solution:We know for the given quadratic equation the value of $a = -2$, $b = 0$ and $c =4$.We have studied that if a quadratic equation does not have the coefficient "b" or the value of "b" is equal to zero and the sign of coefficient "a" and "b" are the same as well, then it will not have a real solution. But in this case, the sign of "a" and "b" are opposite, so this equation should have real roots.$b = 0$$4ac = 4 (-2)(4) = -32$$b^{2}- 4ac = 0 – (-32) = 32$.As the value of the discriminant is positive, it is the second indicator that tells us that this quadratic equation will have real roots. Let us put the value of a, b, and c in the quadratic formula and solve for the roots to verify.$x = \pm\dfrac{ \sqrt{32 }}{2(-2)}$$x = \pm \sqrt{2}$Hence, we have proved the equation has real roots.Example 3: Will the quadratic equation $-2x^{2}- 4 = 0$ have real roots or not?Solution:We can tell by just looking at the equation that it is a no real roots.We know for the given quadratic equation the value of $a = -2$, $b = 0$ and $c = – 2$.As discussed earlier, if the value of $b = 0$ and "a" and "b" have same sign, then there will be no real roots for the given equation and this equation fulfills all the criteria.$b = 0$$4ac = 4 (-2)(-4) = 32$$b^{2}- 4ac = 0 – (32) = -32$.As the value of the discriminant is negative, it is the second indicator that this quadratic equation will not have real roots. Let us put the value of a, b and c in the quadratic formula and solve for the roots to verify.$x = \pm\dfrac{ \sqrt{-32 }}{2(-2)}$$x = \pm \sqrt{2}i$Hence proved the equation has no real rootsExample 4: Solve the quadratic equation $x^{2}+ 5x + 10 = 0$Solution:We know for the given quadratic equation the value of $a =1$, $b = 5$ and $c = 10$The value of $b^{2}= 5^{2}= 25$$4ac = 4 (1)(10) = 40$$b^{2}- 4ac = 25 – 40 = -15$.As the value of discriminant is less than zero, then this equation will not have any real solutions. Let us put the value of a, b and c in quadratic formula and solve for the roots to verify.$x = \dfrac{-5 \pm \sqrt{-15 }}{2(1)}$$x = -2.5 \pm 1.934i$You can verify your answer quickly by using a no-real solution calculator online.How To Write a Quadratic Equation Using the Complex RootsIt is quite easy to write a quadratic equation if you are provided with the complex roots. Suppose we are given the roots of the equation as $4i$ and $-4i$ and we are asked to find the original quadratic equation. We can do that by using the formula $(x-a) (x-b)$ let $a = 4i$ and $b = -4i$.$(x- 4i) (x-(-4i)$$(x-4i) (x+4i)$$x^{2}- 16i^{2}$$x^{2}-16(-1) = x^{2}+ 16$. So the quadratic equation for roots $4i$ and $-4i$ is $x^{2} +16$.Frequently Asked QuestionsWhat Is a Real Solution?A real solution is a solution to an equation that only contains real numbers. In literature, you will often learn that if a quadratic equation discriminant is less than zero, it does not have a solution. It means that it does not have a real solution.What Is a Non-real Solution?A solution that contains imaginary numbers or is written in the form $a+bi$ is called a non-real or complex solution. Here, "a" is real, and the coefficient "b" has iota attached to it, which makes the term imaginary.How Can a Quadratic Equation Have No Solution?The quadratic equation will always have a solution. It will either be real or complex, but there will always be roots for the equation.ConclusionLet us conclude our topic discussion and summarize what we have learned so far.• Quadratic equation will always have a solution, and it can either be real or complex depending upon the value of the discriminant.• There will be no real roots if the value of discriminant is less than zero or $b^{2}-4ac < 0$ or $b^{2} < 4ac$.• When the value of the discriminant is less than zero, we will have two complex solutions and no real rootsAfter studying this guide, we hope you can quickly identify when a quadratic has real solutions and when it only has complex solutions. Copyright © MathBitsNotebook.com. All Rights Reserved. Site by MathBits.com® Quadratics (Equations & Functions) We have seen that the b2 - 4ac portion of the quadratic formula, called the discriminant, can tell us the type of roots of a quadratic equation. The quadratic formula can also give us information about the relationship between the roots and the coefficient of the second term and the constant of the equation itself. Consider the following: Given a quadratic equation: ax2 + bx + c = 0 By the quadratic formulas, the two roots can be represented as Sum of the Roots, r1 + r2: Product of the Roots, r1 • r2: The sum of the roots of a quadratic equation is equal to the negation of the coefficient of the second term, divided by the leading coefficient. The product of the roots of a quadratic equation is equal to the constant term (the third term), divided by the leading coefficient. You will discover in future courses, that these types of relationships also extend to equations of higher degrees. We can also see these relationships emerge from factoring the quadratic equation. The roots will be represented as r1 and r2. Starting with ax2 + bx + c = 0; a ≠ 0 Create a leading coefficient of 1: Express the factors of the equation: Multiply and compare the equations: Conclusion: Let's put this new information to work: Yes, this question can be answered by simply multiplying the factors formed by these roots: But, let's put our new formulas to use and apply the relationship between the roots and the coefficients and constants. The sum of the roots is So the coefficient of the second term is The product of the roots is NOTE: The re-posting of materials (in part or whole) from this site to the Internet is copyright violation and is not considered "fair use" for educators. Please read the "Terms of Use". Once you have the quadratic formula and the basics of quadratic equations down cold, it's time for the next level of your relationship with parabolas: learning about their vertex form. Read on to learn more about the parabola vertex form and how to convert a quadratic equation from standard form to vertex form. feature image credit: SBA73/Flickr Why Is Vertex Form Useful? An Overview The vertex form of an equation is an alternate way of writing out the equation of a parabola. Normally, you'll see a quadratic equation written as ax^2+bx+c, which, when graphed, will be a parabola. From this form, it's easy enough to find the roots of the equation (where the parabola hits the x-axis) by setting the equation equal to zero (or using the quadratic formula). If you need to find the vertex of a parabola, however, the standard quadratic form is much less helpful. Instead, you'll want to convert your quadratic equation into vertex form. What Is Vertex Form? While the standard quadratic form is $ax^2+bx+c=y$, the vertex form of a quadratic equation is $\bi y=\bi a(\bi x-\bi h)^2+ \bi k$. In both forms, y is the y-coordinate, x is the x-coordinate, and a is the constant that tells you whether the parabola is facing up ($+a$) or down ($-a$). (I think about it as if the parabola was a bowl of applesauce; if there's a $+a$, I can add applesauce to the bowl; if there's a $-a$, I can shake the applesauce out of the bowl.) The difference between a parabola's standard form and vertex form is that the vertex form of the equation also gives you the parabola's vertex: (h,k). For example, take a look at this fine parabola, $y=3(x+4/3)^2-2$: Based on the graph, the parabola's vertex looks to be something like (-1.5,-2), but it's hard to tell exactly where the vertex is from just the graph alone. Fortunately, based on the equation $y=3(x+4/3)^2-2$, we know the vertex of this parabola is $(-4/3,-2)$. Why is the vertex $(-4/3,-2)$ and not $(4/3,-2)$ (other than the graph, which makes it clear both the x- and y-coordinates of the vertex are negative)? Remember: in the vertex form equation, h is subtracted and k is added. If you have a negative h or a negative k, you'll need to make sure that you subtract the negative h and add the negative k. In this case, this means: $y=3(x+4/3)^2-2=3(x-(-4/3))^2+(-2)$ and so the vertex is $(-4/3,-2)$. You should always double-check your positive and negative signs when writing out a parabola in vertex form, particularly if the vertex does not have positive x and y values (or for you quadrant-heads out there, if it's not in quadrant I). This is similar to the check you'd do if you were solving the quadratic formula ($x={-b±√{b^2-4ac}}/{2a}$) and needed to make sure you kept your positive and negatives straight for your as, bs, and cs. Below is a table with further examples of a few other parabola vertex forms, along with their vertices. Note in particular the difference in the $(x-h)^2$ part of the parabola vertex form equation when the x coordinate of the vertex is negative. Parabola Vertex Form Vertex Coordinates $y=5(x-4)^2+17$ $(4,17)$ $y=2/3(x-8)^2-1/3$ $(8,-1/3)$ $y=144(x+1/2)^2-2$ $(-1/2,-2)$ $y=1.8(x+2.4)^2+2.4$ $(-2.4,2.4)$ How to Convert From Standard Quadratic Form to Vertex Form Most of the time when you're asked to convert quadratic equations between different forms, you'll be going from standard form (ax^2+bx+c) to vertex form ($a(x-h)^2+k$). The process of converting your equation from standard quadratic to vertex form involves doing a set of steps called completing the square. (For more about completing the square, be sure to read this article.) Let's walk through an example of converting an equation from standard form to vertex form. We'll start with the equation $y=7x^2+42x-3/14$. The first thing you'll want to do is move the constant, or the term without an x or x^2 next to it. In this case, our constant is $-3/14$. (We know it's negative $3/14$ because the standard quadratic equation is ax^2+bx+c, not ax^2+bx-c.) First, we'll take that $-3/14$ and move it over to the left side of the equation: $y+3/14=7x^2+42x$ The next step is to factor out the 7 (the a value in the equation) from the right side, like so: $y+3/14=7(x^2+6x)$ Great! This equation is looking much more like vertex form, $y=a(x-h)^2+k$! At this point, you might be thinking, "All I need to do now is move the $3/14$ back over to the right side of the equation, right?" Alas, not so fast. If you take a look at part of the equation inside of the parentheses, you'll notice a problem: it's not in the form of $(x-h)^2$. There are too many xs! So we're not quite done yet. What we need to do now is the hardest part—completing the square. Let's take a closer look at the x^2+6x part of the equation. In order to factor (x^2+6x) into something resembling $(x-h)^2$, we're going to need to add a constant to the inside of the parentheses—and we're going to need to remember to add that constant to the other side of the equation (since the equation needs to stay balanced). To set this up (and make sure we don't forget to add the constant to the other side of the equation), we're going to create a blank space where the constant will go on either side of the equation: $y+3/14+7($ $)=7(x^2+6x+$ $)$ Note that on the left side of the equation, we made sure to include our a value, 7, in front of the space where our constant will go; this is because we're not just adding the constant to the right side of the equation, but we're multiplying the constant by whatever is on the outside of the parentheses. (If your a value is 1, you don't need to worry about this.) The next step is to complete the square. In this case, the square you're completing is the equation inside of the parentheses—by adding a constant, you're turning it into an equation that can be written as a square. To calculate that new constant, take the value next to x (6, in this case), divide it by 2, and square it. $(6/2)^2=(3)^2=9$. The constant is 9. The reason we halve the 6 and square it is that we know that in an equation in the form $(x+p)(x+p)$ (which is what we're trying to get to), $px+px=6x$, so $p=6/2$; to get the constant p^2, we thus have to take $6/2$ (our p) and square it. Now, replace the blank space on either side of our equation with the constant 9: $y+3/14+7(9)=7(x^2+6x+9)$ $y+{3/14}+63=7(x^2+6x+9)$ $y+{3/14}+{882/14}=7(x^2+6x+9)$ $y+{885/14}=7(x^2+6x+9)$ Next, factor the equation inside of the parentheses. Because we completed the square, you will be able to factor it as $(x+{\some umber})^2$. $y+{885/14}=7(x+3)^2$ Last step: move the non-y value from the left side of the equation back over to the right side: $y=7(x+3)^2-{885/14}$ Congratulations! You've successfully converted your equation from standard to vertex form. Now, most problems won't just ask you to convert your equations from standard form to vertex form; they'll want you to actually give the coordinates of the vertex of the parabola. To avoid getting tricked by sign changes, let's write out the general vertex form equation directly above the vertex form equation we just calculated: $y=a(x-h)^2+k$ $y=7(x+3)^2-{885/14}$ And then we can easily find h and k: $-h=3$ $h=-3$ $+k=-{885/14}$ The vertex of this parabola is at coordinates $(-3,-{885/14})$. Whew, that was a lot of shuffling numbers around! Fortunately, converting equations in the other direction (from vertex to standard form) is a lot simpler. How To Convert From Vertex Form to Standard Form Converting equations from their vertex form to the regular quadratic standard form is a much more straightforward process: all you need to do is multiply out the vertex form. Let's take our example equation from earlier, $y=3(x+4/3)^2-2$. To turn this into standard form, we just expand out the right side of the equation: $$y=3(x+4/3)^2-2$$ $$y=3(x+4/3)(x+4/3)-2$$ $$y=3(x^2+{8/3}x+16/9)-2$$ $$y=3x^2+8x+{16/3}-2$$ $$y=3x^2+8x+{16/3}-{6/3}$$ $$ y=3x^2+8x+10/3$$ Tada! You've successfully converted $y=3(x+4/3)^2-2$ to its ax^2+bx+c form. Parabola Vertex Form Practice: Sample Questions To wrap up this exploration of vertex form, we have four example problems and explanations. See if you can solve the problems yourself before reading through the explanations! #1: What is the vertex form of the quadratic equation $x^2+ 2.6x+1.2$? #2: Convert the equation $7y=91x^2-112$ into vertex form. What is the vertex? #3: Given the equation $y=2(x-3/2)^2-9$, what are the x-coordinates of where this equation intersects with the x-axis? #4: Find the vertex of the parabola $y=({1/9}x-6)(x+4)$. Parabola Vertex Form Practice: Solutions #1: What is the vertex form of the quadratic equation ${\bi x}^2+2.6\bi x+1.2$? Starting by separating out the non-x variable onto the other side of the equation: $y-1.2=x^2+2.6x$ Since our a (as in ax^2+bx+c) in the original equation is equal to 1, we don't need to factor it out of the right side here (although if you want, you can write $y-1.2=1(x^2+2.6x)$). Next, divide the x coefficient (2.6) by 2 and square it, then add the resulting number to both sides of the equation: $(2.6/2)^2=(1.3)^2=1.69$ $y-1.2+1(1.69)=1(x^2+2.6x+1.69)$ Factor the right side of the equation inside the parentheses: $y-1.2+1.69=(x+1.3)^2$ Finally, combine the constants on the left side of the equation, then move them over to the right side. $y+0.49=(x+1.3)^2$ Our answer is $y=(x+1.3)^2-0.49$. #2: Convert the equation $7\bi y=91\bi x^2-112$ into vertex form. What is the vertex? When converting an equation into vertex form, you want the y have a coefficient of 1, so the first thing we're going to do is divide both sides of this equation by 7: $7y=91x^2-112$ ${7y}/7= {91x^2}/7-112/7$ $y=13x^2-16$ Next, bring the constant over to the left side of the equation: $y+16=13x^2$ Factor out the coefficient of the x^2 number (the a) from the right side of the equation: $y+16=13(x^2)$ Now, normally you'd need to complete the square on the right side of the equation inside the parentheses. However, x^2 is already a square, so you don't need to do anything besides moving the constant from the left side of the equation back to the right side: $y=13(x^2)-16$ To find the vertex: $y=a(x-h)^2+k$ $y=13(x^2)-16$ $-h=0$, so $h=0$ $+k=-16$, so $k=-16$ The vertex of the parabola is at $(0, -16)$. #3: Given the equation $\bi y=2(\bi x-3/2)^2-9$, what is(are) the $\bi x$-coordinate(s) of where this equation intersects with the $\bi x$-axis? Because the question is asking you to find the x-intercept(s) of the equation, the first step is to set $y=0$. $y=0=2(x-3/2)^2-9$. Now, there are a couple of ways to go from here. The sneaky way is to use the fact that there's already a square written into the vertex form equation to our advantage. First, we'll move the constant over to the left side of the equation: $0=2(x-3/2)^2-9$ $9=2(x-3/2)^2$ Next, we'll divide both sides of the equation by 2: $9/2=(x-3/2)^2$ Now, the sneaky part. Take the square root of both sides of the equation: $√(9/2)=√((x-3/2)^2)$ $±3/√2=(x-3/2)$$± home / algebra / solving equations / vertex formVertex form is another form of a quadratic equation. The standard form of a quadratic equation is ax2 + bx + c. The vertex form of a quadratic equation is a(x - h)2 + k where a is a constant that tells us whether the parabola opens upwards or downwards, and (h, k) is the location of the vertex of the parabola. The following are two examples of quadratic equations written in vertex form: 2(x - 7)2 + 3; vertex at (7, 3) 2(x + 7)2 - 3; vertex at (-7 , -3) The above examples show that we can't just read off the values based on their position in the equation. We need to remember the vertex form a(x - h)2 + k. If, like in equation (1.) above, the signs in the equation match that of the generalized vertex form, then we can read off (h, k) as the vertex. However, if, like in equation (2.), the signs are different from those in the general vertex form equation, we need to take the signs into account; for h, the sign of the x-coordinate of the vertex is opposite of that in the vertex form equation; for k, the sign of the y-coordinate is the same as that in the vertex form equation. This is due to the nature of positive/negative numbers. Converting from standard form to vertex form Converting a quadratic equation from standard form to vertex form involves a technique called completing the square. Refer to the completing the square page for a detailed explanation. Generally, it involves moving the constant to the other side of the equation and finding a constant that allows us to write the right hand side of the equation in a form resembling vertex form, applying that constant to the left side of the equation, then shifting the constant on the left side back to the right side. Example Convert y = 3x2 + 9x + 4 to vertex form: y - 4 = 3x2 + 9x y - 4 = 3(x2 + 3x) y - 4 + 3(?) = 3(x2 + 3x + ?) y - 4 + 3() = 3(x2 + 3x +) y - = 3(x +)2 y = 3(x +)2 - Converting from vertex form to standard form To convert from vertex form to standard form, we simply expand vertex form. We can confirm that our above equation in vertex form is the same as the original equation in standard form by expanding it: y = 3(x +)2 - y = 3(x2 + x + x + ()2) - y = 3(x2 + 3x +) - y = 3x2 + 9x + - y = 3x2 + 9x + 4 Expanding our equation in vertex form yields the same equation we started with in standard form, so we've confirmed that our conversion to vertex form was correct. Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Examples, videos, and solutions to help Algebra I students learn how to graph simple quadratic equations of the form y = a(x-h)2 + k (completed-square or vertex form), recognizing that (h, k) represents the vertex of the graph and use a graph to construct a quadratic equation in vertex form. Students understand the relationship between the leading coefficient of a quadratic function and its concavity and slope and recognize that an infinite number of quadratic functions share the same vertex. New York State Common Core Math Algebra I, Module 4, Lesson 16 Worksheets for Algebra 1 Lesson 16 Summary When graphing a quadratic equation in vertex form, y=a(x-h)2 + k, (h, k) are the coordinates of the vertex. Opening Exercise Graph the equations y = x2, y = (x - 2)2, and y = (x + 2)2 on the interval -3 ≤ x ≤ 3. Exercises 1-2 Without graphing, state the vertex for each of the following quadratic equations. a. y = (x - 5)2 + 3 b. y = x2 - 2.5 c. y = (x + 4)2 Write a quadratic equation whose graph will have the given vertex. a. (1.9, -4) b. (0, 100) c. (-2, 3/2) Example 1 Caitlin has 60 feet of material that can be used to make a fence. Using this material, she wants to create a rectangular pen for her dogs to play in. What dimensions will maximize the area of the pen? a. Let w be the width of the rectangular pen in feet. Write an expression that represents the length when the width is feet. b. Define a function A(w) that describes the area , in terms of the width, w. c. Rewrite A(w) in vertex form. d. What are the coordinates of the vertex? Interpret the vertex in terms of the problem. e. What dimensions maximize the area of the pen? Do you think this is a surprising answer? Show Step-by-step Solutions Try out our new and fun Fraction Concoction Game. Add and subtract fractions to make exciting fraction concoctions following a recipe. There are four levels of difficulty: Easy, medium, hard and insane. Practice the basics of fraction addition and subtraction or challenge yourself with the insane level. We welcome your feedback, comments and questions about this site or page. Please submit your feedback or enquiries via our Feedback page. When written in "vertex form": • (h, k) is the vertex of the parabola, and x = h is the axis of symmetry. • the h represents a horizontal shift (how far left, or right, the graph has shifted from x = 0). • the k represents a vertical shift (how far up, or down, the graph has shifted from y = 0). • notice that the h value is subtracted in this form, and that the k value is added. If the equation is y = 2(x - 1)2 + 5, the value of h is 1, and k is 5. If the equation is y = 3(x + 4)2 - 6, the value of h is -4, and k is -6. To Convert from f (x) = ax2 + bx + c Form to Vertex Form: Method 1: Completing the Square To convert a quadratic from y = ax2 + bx + c form to vertex form, y = a(x - h)2+ k, you use the process of completing the square. Let's see an example. Convert y = 2x2 - 4x + 5 into vertex form, and state the vertex. Equation in y = ax2 + bx + c form. y = 2x2 - 4x + 5 Since we will be "completing the square" we will isolate the x2 and x terms ... so move the + 5 to the other side of the equal sign. y - 5 = 2x2 - 4x We need a leading coefficient of 1 for completing the square ... so factor out the current leading coefficient of 2. y - 5 = 2(x2 - 2x) Get ready to create a perfect square trinomial. BUT be careful!! In previous completing the square problems with a leading coefficient not 1, our equations were set equal to 0. Now, we have to deal with an additional variable, "y" ... so we can never "get rid of " the factored 2. When we add a box in the parentheses, the box will be multiplied by 2 on both sides of the equal sign. Find the perfect square trinomial. Take half of the coefficient of the x-term inside the parentheses, square it, and place it in the box. Simplify and convert the right side to a squared expression. y - 3 = 2(x - 1)2 Isolate the y-term ... so move the -3 to the other side of the equal sign. y = 2(x - 1)2 + 3 In some cases, you may need to transform the equation into the "exact" vertex form of y = a(x - h)2 + k, showing a "subtraction" sign in the parentheses before the h term, and the "addition" of the k term. (This was not needed in this problem.) y = 2(x - 1)2 + 3 Vertex form of the equation. Vertex = (h, k) = (1, 3) (The vertex of this graph will be moved one unit to the right and three units up from (0,0), the vertex of its parent y = x2.) Here's a sneaky, quick tidbit: When working with the vertex form of a quadratic function, and . The "a" and "b" referenced here refer to f (x) = ax2 + bx + c. Method 2: Using the "sneaky tidbit", seen above, to convert to vertex form: y = ax2 + bx + c form of the equation. y = 2x2 - 4x + 5 Find the vertex, (h, k). and . [f (h) means to plug your answer for h into the original equation for x.] a = 2 and b = -4 Vertex: (1,3) Write the vertex form. y = a(x - h)2 + k y = 2(x - 1)2 + 3 To Convert from Vertex Form to y = ax2 + bx + c Form: Simply multiply out and combine like terms: y = 2(x - 1)2 + 3 y = 2(x2 - 2x + 1) + 3 y = 2x2 - 4x + 2 + 3 y = 2x2 - 4x + 5 3 Graphing a Quadratic Function in Vertex Form: 1. Start with the function in vertex form: y = a(x - h)2 + k y = 3(x - 2)2 - 4 2. Pull out the values for h and k. If necessary, rewrite the function so you can clearly see the h and k values. (h, k) is the vertex of the parabola. Plot the vertex. y = 3(x - 2)2 + (-4) h = 2; k = -4 Vertex: (2, -4) 3. The line x = h is the axis of symmetry. Draw the axis of symmetry. x = 2 is the axis of symmetry 4. Find two or three points on one side of the axis of symmetry, by substituting your chosen x-values into the equation. For this problem, we chose (to the left of the axis of symmetry): x = 1; y = 3(1 - 2)2 - 4 = -1 x = 0; y = 3(0 - 2)2 - 4 = 8 Plot (1, -1) and (0,8) 5. Plot the mirror images of these points across the axis of symmetry, or plot new points on the right side. Remember, when drawing the parabola to avoid "connecting the dots" with straight line segments. A parabola is curved, not straight, as its slope is not constant.